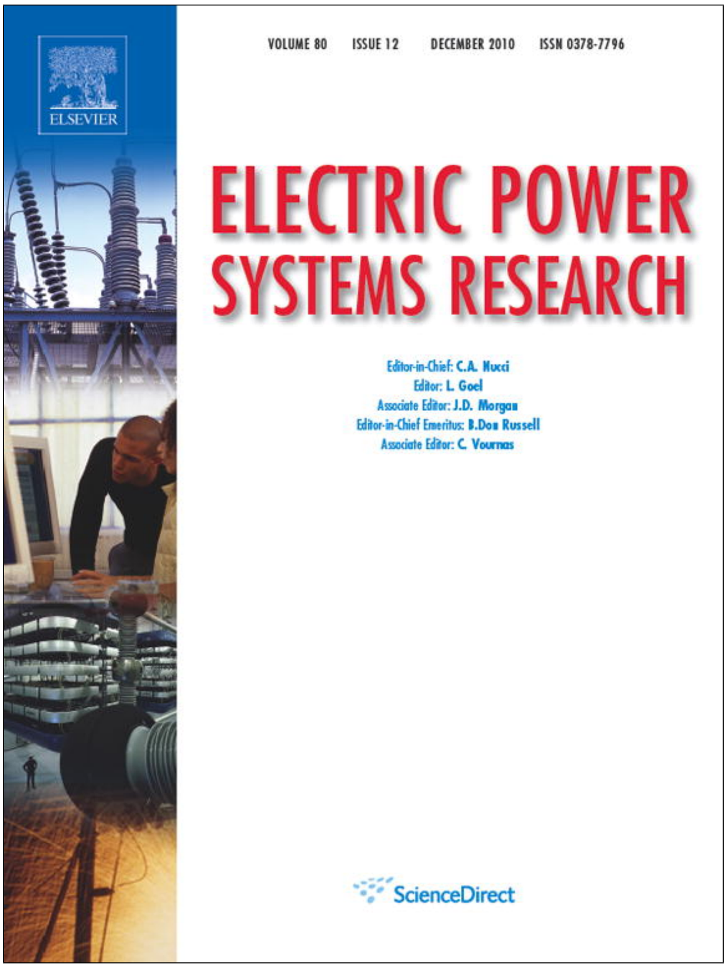
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Robust decentralized PID-based power system stabilizer design using an ILMI approach

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ABSTRACT

Thanks to its essential functionality and structure simplicity, proportional-integral-derivative (PID) controllers are commonly used by industrial utilities. A robust PID-based power system stabilizer (PSS) is proposed to properly function over a wide range of operating conditions. Uncertainties in plant parameters, due to variation in generation and load patterns, are expressed in the form of a polytopic model. The PID control problem is firstly reduced to a generalized static output feedback (SOF) synthesis. The derivative action is designed and implemented as a high-pass filter based on a low-pass block to reduce its sensitivity to sensor noise. The proposed design algorithm adopts a quadratic Lyapunov approach to guarantee α-decay rate for the entire polytope. A constrained structure of Lyapunov function and SOF gain matrix is considered to enforce a decentralized scheme. Setting of controller parameters is carried out via an iterative linear matrix inequality (ILMI). Simulation results, based on a benchmark model of a two-area four-machine test system, are presented to compare the proposed design to a well-tuned conventional PSS and to the standard IEEE-PSS4B stabilizer.

1. Introduction

Since power systems are highly nonlinear and are often subjected to small and large disturbances, conventional fixed-parameter PSS may fail to maintain stability or lead to a degraded performance [1–3]. Two main approaches, namely adaptive control [4–8] and robust control [9–15] have been proposed to enhance the performance of a PSS. The implementation of an adaptive controller needs tough precautions to assure the persistent excitation conditions. Adaptive controllers generally have poor performance during the learning phase unless they are appropriately initialized [5]. Robust control provides an effective approach to handle uncertainties introduced by continuous variation in generation and load patterns. Many papers addressed robust PSS design via different control approaches. The H_∞ approach is applied to the design of a robust PSS for a single-machine infinite-bus system in [15]. In this approach, the uncertainty in the plant parameters is captured in terms of bounds on the frequency response. Also, Kharitonov theorem is applied to the design of a robust PSS for a single-machine infinite-bus system in [10–16]. Recently, many papers addressed this problem via an LMI approach [11–14]. In [11], robust pole placement via state feedback PSS design is presented. However, this design approach assumes full state availability. Werner et al. [12] presented the model uncertainty as a linear fractional transformation (LFT) and designed an output feedback PSS that guarantees stability for all admissible plants, while minimizing a quadratic performance index for a nominal plant which is not a practical operating point. The resulting controller has the same order as that of the plant and the case of multimachine is not considered. Ramos et al. [13] used a combination of LMI technique and direct feedback linearization to get a robust centralized dynamic output feedback PSS of order 12 to suppress inter-area oscillations. In [14], robust decentralized PSS design problem is expressed as minimizing a linear objective function under LMI and bilinear matrix inequality (BMI) constraints. The authors in [14] also reported the problem of designing reduced-order decentralized H_∞ dynamic output feedback PSS based on parameter continuation method in LMI framework. This design approach suffers from the non-convexity of BMIs.

Due to enlarged scale of power systems, many efforts have been devoted to decentralized robust excitation control strategies [17–22]. A linear feedback control law based on the solution of parameterized Riccati equations for each subsystem was developed in [23]. These ideas have been applied to the design of a robust exciter based on direct feedback linearization, which transforms the original nonlinear system to a linear one. This transformation makes the design procedure simple, but the controller implementation becomes complex as it is of a nonlinear nature. There also several results presented in [24,25], where a linear feedback is

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based on appropriately chosen Lyapunov functions to produce lower bounds on local gains. This method incorporates a quadratic term in the model which cannot be properly used in analysis.

PID controllers are commonly used in many industrial applications thanks to its essential functionality and structural simplicity. Systematic methods for designing PID controllers have been extensively studied in the literature, including robust design of PID controllers [26,27]. Design of multivariable PID controllers via LMI is addressed under the topic of static output feedback (SOF) stabilization. Zheng et al. [28] presented an approach to transform the problem of PID controller design to that of SOF controller design. The transformation in [28] requires the inevitability of a certain respective matrix and additional computation is needed to recover the PID gain matrices. In our proposed approach, PID controller design is transferred to that of SOF with no additional transformation and computation. Further, the problem of derivative term is considered by including a high-pass filter in the SOF design problem. Moreover, once the SOF problem is solved PID gains are ready in hand and no additional computation is needed. The SOF stabilization problem is extensively addressed in the LMI framework, e.g. [29–36]. The authors of [36] presented a recent and an elegant ILMI algorithm to solve the SOF stabilization problem for nominal plants.

This paper proposes two ILMI algorithms to solve the problem of quadratic stabilizability of polytopic systems via SOF controllers. Moreover, a constrained structure of Lyapunov function and the SOF gain matrix is proposed to have a set of decentralized controllers. Uncertainty about the plant operating point is expressed as a polytopic system. The proposed design guarantees α-stability of different operating points inside this polytope. The proposed design considers the sensitivity of the derivative term to sensor noise and improves it by inclusion of a low-pass filter in the design procedure.

The rest of the paper is organized as follows. Section 2 presents a review of SOF stabilization problem via ILMI algorithms. The proposed modifications of the algorithms, presented in [36] to meet quadratic stabilizability of polytopic system and the decentralized scheme, tags this section. The polytopic modeling of power systems and PID-based PSS design algorithm are presented in Section 3. Simulation results and comparison of the proposed design to a well-tuned conventional PSS [1] and to the standard IEEE-PSS4B stabilizer [38] are depicted in Section 4. Section 5 concludes this work.

2. Static output feedback (SOF) stabilization

2.1. A review of SOF stabilization—some results revisited

Throughout the paper, the notation $X>0$(respectively$X\geq 0$) means that X is symmetric positive-definite (respectively semidefinite) matrix. Consider the following LTI continuous time system:

$$\Gamma:\ \dot{x}=Ax+Bu,\quad y=Cx \tag{1}$$

where $x\in R^n$ is the state vector, $u\in R^r$ is the control vector and $y\in R^m$ is the vector of the measured outputs, A, B and C are constant matrices. The system Γ may be identified by its realization (A, B, C). All systems considered are assumed to be stablizable via SOF gain. The SOF stabilization problem is to find a static output feedback law: $u=Fy$, where $F\in R^{r\times m}$, such that the closed loop system Γ_c given by:

$$\Gamma_c:\ \dot{x}=(A+BFC)x \tag{2}$$

is Hurwitz-stable, i.e. has all its roots in the open left-half of the complex s-plane. Also, Γ_c is stable if and only if there exists $P=P^T>0$ such that

$$P(A+BFC)+(A+BFC)^T\quad P<0 \tag{3}$$

Definition 1. Γ is said to be stablizable via SOF if there exists F such that Γ_c is Hurwitz-stable.

Definition 2. Γ is said to be α-stablizable via SOF if Γ_c has its eigenvalues in the strict left-hand side of the line $s=\alpha/2$ in the complex s-plane, i.e.:

$$P(A+BFC)+(A+BFC)^TP-\alpha P<0 \tag{4}$$

Conditions (3) and (4) are BMIs which are not convex optimal problems. An ILMI method was proposed in [29] to solve this SOF problem, where an additional variable X was introduced to such that the stability condition becomes significant when $X\neq P$. This algorithm tried to find a sequence of the additional variables such that the sufficient condition is close to the necessary and sufficient one. The main optimization problem in this algorithm was an ill-posed generalized eigenvalues problem (GEVP). A similar idea developed in [32] is used in the so-called substitutive LMI method. Recently, an improved ILMI algorithm for SOF stabilization without introducing any additional variables is proposed in [36]. The authors of [36] presented two separate ILMI algorithms. The first algorithm is used to get suitable initial variables. The second ILMI algorithm uses these initial variables to find the SOF gain matrix. If these initial variables cannot be found by the first algorithm, the SOF stabilization problem may not have solutions.

2.2. SOF stabilization of polytopic systems

Simultaneous stabilization is an important issue in the area of robust control design. It is the problem of determining a single controller which will simultaneously stabilize a finite collection of plants. It is usually applied to linear plants characterized by different modes or to the stabilization of nonlinear plants linearized at different operating points. Simultaneous stabilization via SOF is addressed in [30]. Cao et al. [30] presented some necessary and sufficient LMI conditions for simultaneous stabilizability of τ strictly proper MIMO plants via SOF and state feedback in the form of coupled algebraic Riccati inequalities (ARIs). This approach utilizes a heuristic iterative algorithm based on LMI to solve the coupled inequalities. However, our paper proposes a simpler approach by extending the algorithms presented in [36] to solve simultaneous stabilization problem via SOF control.

Consider the system in (1), further consider that $(A, B, C)\in\Omega$, where,

$$\Omega=Co\{(A^1,B^1,C^1),(A^2,B^2,C^2),\ldots,(A^N,B^N,C^N)\} \tag{5}$$

Ω is assumed a convex polytope of matrices for which each element may be expressed as a convex combination of the N vertices of Ω, i.e.:

$$(A(\varsigma),B(\varsigma),C(\varsigma))=\sum_{i=1}^{N}\varsigma_i(A^i,B^i,C^i) \tag{6}$$

$$\varsigma\in\Xi=\left\{\varsigma\in[0,\infty)^N:\sum_{i=1}^{N}\varsigma_i=1\right\} \tag{7}$$

The objective of the SOF is to find a robustly stabilizing control law $u(t)=Fy(t)$ for the model (5), (6) and (7), i.e. to find a single gain matrix $F\in R^{r\times m}$ such that every vertex of the polytope Ω_c is Hurwitz-stable.

$$\Omega_c=Co\{(A^1+B^1FC^1),\ldots,(A^N+B^NFC^N)\} \tag{8}$$

Robust stability of this polytope is satisfied if there exist matrices $P_i=P_i^T>0$ such that

$$P^i(A^i+B^iFC^i)+(A^i+B^iFC^i)^TP^i<0,\quad i=1,2,\ldots,N \tag{9}$$

Quadratic stability of the polytope (5) is enforced if a unique positive-definite matrix is found for all vertices of the polytope, i.e. $P^i = P,\ i = 1, \ldots, N$.

The algorithms in [36] are modified to match the case of quadratic stabilizability of polytopic systems. It should be noted that matrix superscripts are used to indicate index of the polytope vertices, while matrix subscripts are used to indicate the iteration index in the following algorithms.

Algorithm 1 *(Initialization algorithm).*

step (1) Set $i = 1$, $P_0 = I$ and $L_0 = I$.
step (2) Derive a P_i and L_i by solving the following optimization problem for P_i, L_i, V_1 and V_2:OP1: Minimize $trace\,(P_iL_{i-1} + L_iP_{i-1})$ subject to the following LMI constraints:

$$P_iA^j + A^{j^T}P_i + V_1C^j + C^{j^T}V_1^T < 0, \quad j = 1, \ldots, N \tag{10}$$

$$A^jL_i + L_iA^{j^T} + B^jV_2 + V_2^TB^{j^T} < 0, \quad j = 1, \ldots, N \tag{11}$$

$$(12)\begin{bmatrix} P_i & I \\ I & L_i \end{bmatrix} \geq 0$$

step (3) If $Trace(P_iL_i) - n < \varepsilon_1$, an initial P is found, stop, where ε_1 is a prescribed tolerance.
step (4) If $Trace(P_iL_i) - Trace(P_{i-1}L_{i-1}) < \varepsilon_2$, an initial P may not be found, stop, where ε_2 is a prescribed tolerance.
step (5) Set $i = i + 1$, go to step 2.

This algorithm guarantees that an initial common positive-definite matrix is to be found at all vertices of the polytope.

Algorithm 2 *(Find an α_r-decay rate robust SOF controller gain matrix).*

step (1) Set $i = 1$ and $P_1 = P$ as obtained from Algorithm 1.
step (2) Solve the following optimization problem for F with given P_i.OP1: minimize α_2 subject to the following LMI constraint:

$$P_i(A^j + B^jFC^j) + (A^j + B^jFC^j)^TP_i - \alpha_iP_i < 0, \quad j = 1, \ldots, N \tag{13}$$

step (3) If $\alpha_i \leq \alpha_r$, F is a stabilizing SOF gain, stop
step (4) Set $i = i + 1$, solve the following optimization problem for Pi with given F.OP2: minimize α_i subject to (13)
step (5) If $\alpha_i \leq \alpha_r$, F is a stabilizing SOF gain, stop.
step (6) Solve the following optimization problem for P_i with given F and α_i, set $k = 1$.OP3: minimize $Trace(P_i)$ subject to (13). If trace minimization problem has an infeasible solution, set $k = k + 1$ and put $\alpha_i = \alpha_i/k$ if $\alpha_i < 0$ or put $\alpha_i = \alpha_i \times k$ if $\alpha_i > 0$ until a feasible solution is reached.
step (7) If $\left\|P_i - P_{i-1}\right\| / \left\|P_i\right\| < \delta$, where δ is a prescribed tolerance, go to step 8, else set $i = i + 1$ and $P_i = P_{i-1}$ and then go to step 2.
step (8) The system may not be stabilizable via SOF, stop.

The proposed algorithms differ from that presented in [36] in the following points:

i. LMI constraints (10), (11) and (13) must be enforced at all vertices of the polytopic system using common positive-definite matrices.
ii. Stopping criterion in Algorithm 2 is changed to $\alpha_i \leq \alpha_r$, in order to guarantee the desired minimum decay rate at all vertices of the polytope.
iii. To overcome the infeasibility of trace minimization in Algorithm 2, the following procedure is proposed to get a feasible solution. Set $k = k + 1$ and put $\alpha_i = \alpha_i/k$ if $\alpha_i < 0$ or put $\alpha_i = \alpha_i \times k$ if $\alpha_i > 0$ until a feasible solution is reached. This problem is not reported in [36].
iv. An extension to the case of decentralized SOF controllers design is investigated.

The proposed Algorithms 1 and 2 comprise a set of LMI constraints which belongs to the generic problems of LMI, namely optimization and generalized eigenvalue problem (GEVP). LMI control toolbox, for use with MATLAB [37], was used to solve LMI constraints presented in this paper. The solution of LMI problems presented in Algorithms 1 and 2 depends mainly on two functions of the LMI control toolbox, namely mincx and gevp [37]. The syntax of an LMI problem using LMI control toolbox, for use with MATLAB, is given for illustration. Consider the LMI problem in step 4 of Algorithm 2; the syntax of this problem in the framework of LMI control toolbox is given as follows:

```
setlmis([]);
Pi = lmivar(1,[ns,1]);
lmiterm([1 1 1 0],0);
lmiterm([−1 1 1 Pi],1,1);   %   P > 0
for j = 1:nsys
   sj = psinfo(sys,'sys',j);
   [aj,bj,cj,dj] = ltiss(sj);
   lmiterm([(j + 1) 1 1 Pi],(aj + bj*F*cj)',1,'s');
   lmiterm([−(j + 1) 1 1 Pi],1,1);
end
lmis = getlmis;
 [alfa_min,xopt] = gevp(lmis,nsys,opt); % generalized eigenvalue problem solver
Pi = dec2mat(lmis,xopt,Pi);
```

2.3. Decentralized SOF stabilization of multimachine power systems

The analysis in this section is extended to a multimachine power system comprising m-machines whose linearized model takes the form of (1). Further, the dynamics of Machine i in a multimachine power system take the following state-space realization:

$$\begin{aligned} \dot{x}_i &= A_{ii}x_i + \sum_{j=1, j\neq i}^{m} A_{ij}x_j + B_iu_i \\ y_i &= C_ix_i \end{aligned} \tag{14}$$

The term $\sum_{j=1, j\neq i}^{m} A_{ij}x_j$ represents the interaction of machine i with all other machines in the considered system. Each machine subsystem is both input and output decentralized, hence, it is axiom to consider a control law of the form:

$$u_i = F_iy_i \quad i = 1, \ldots, m \tag{15}$$

The closed loop subsystem takes the following form:

$$\dot{x}_i = (A_{ii} + B_iF_iC_i)x_i + \sum_{j=1, j\neq i}^{m} A_{ij}x_j$$

The overall closed loop power system takes the form of

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_m \end{bmatrix} = \begin{bmatrix} A_{11} + B_1F_1C_1 & A_{12} & \cdots & A_{1m} \\ A_{21} & A_{22} + B_2F_2C_2 & \cdots & A_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ A_{m1} & A_{m2} & \cdots & A_{mm} + B_mF_mC_m \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{bmatrix} \tag{16}$$

This form can be rewritten as follows:

$$\dot{x} = (A + BFC)x,$$

where

$$B = \bigoplus_{i=1}^{m} B_i = \begin{bmatrix} B_1 & & & \\ & B_2 & & \\ & & \ddots & \\ & & & B_m \end{bmatrix}, \quad C = \bigoplus_{i=1}^{m} C_i, \quad \text{and}, \quad F = \bigoplus_{i=1}^{m} F_i \tag{17}$$

The objective in this subsection is to find a robust decentralized SOF controller that guarantees α-stability of N different operating points. To consider the problem of robust decentralized SOF controller design, Algorithms 1 and 2 must enforce a constrained structure of Lyapunov and controller matrices as follows:

$$P = \bigoplus_{i=1}^{m} P_i, \quad L = \bigoplus_{i=1}^{m} L_i, \quad V_1 = \bigoplus_{i=1}^{m} V, \quad \text{and}, \quad V_2 = \bigoplus_{i=1}^{m} V_{2i} \tag{18}$$

3. PID-based PSS design

This section describes the steps involved in robust PID-based PSS design. Firstly, the power system model is prepared to initiate the design procedure.

3.1. The polytopic model for power systems

Power system behavior is usually modeled by a set of differential equations representing the dynamics of the generators and their respective automatic voltage regulators (AVRs) coupled to another set of algebraic equations describing the stators, transmission network and the loads that are modeled by constant impedance. Incorporating the algebraic equations in the differential set, by a process known as network reduction, the resulting model is in the traditional state-space form. Classical techniques involved in the design of PSS are based on linearized models. The robustness of these PSSs is limited with respect to continuous variations in the operating point, due to the fact that a linearized model is accurate only in the neighborhood of the operating point around which the system is linearized. A polytopic model is a good alternative to overcome this problem introduced by classical techniques. The system is linearized around different N typical points producing a polytope of matrices as in (5). The convexity of (5) can be explored by simple ways. For example, if certain properties like quadratic stability and/or minimum decay rate are satisfied at all vertices of Ω, they extend to all matrices that are within the polytope. The 4th order model is adopted to represent the dynamics of each machine as given Appendix A.1. For network equations, it is proposed to use the form of bus impedance matrix in the synchronously rotating frame of reference Z_{DQ} after network reduction assuming constant-impedance loads, i.e., $V_{DQ} = Z_{DQ} I_{DQ}$, where V_{DQ} and I_{DQ} are machine terminal voltage and terminal currents referred to a common reference. The machine–network transformation is given in Appendix A.2. Typically, speed deviation measurements are used as the feedback signals. The supplementary signal for each machine based on PID control law takes the following form:

$$u_i = K_p \Delta\omega + K_i \int_0^t \Delta\omega dt + K_d \frac{d\Delta\omega}{dt} \tag{19}$$

The integral action can be rewritten in terms of the incremental power angle. For incremental quantities, it is derived from (24) that:

$$\frac{d\Delta\delta}{dt} = \omega_o \Delta\omega \Rightarrow d\Delta\delta = \omega_o \Delta\omega dt$$

$$\int_0^t \Delta\omega dt = \frac{1}{\omega_o} \int_0^t d\Delta\delta = \frac{1}{\omega_o} \Delta\delta \tag{20}$$

Consequently, the integral action is expressed in terms of the state variable $\Delta\delta$ for the linearized model (small-signal model). PSSs are incorporated in power systems to suppress electromechanical oscillations in the range of 0.2–3 Hz. This specification determines the range in which the derivative term should be active. It is proposed to consider a high-pass filter whose cut-off frequency accounts for this range of desired frequencies. A derivative action is

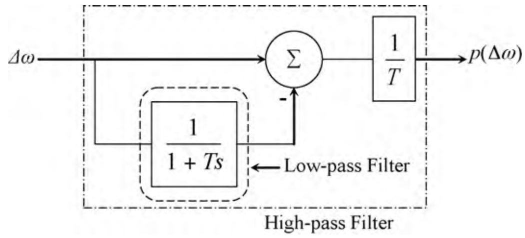


Fig. 1. Derivative action implemented as a high-pass filter based on a low-pass block.

considered as shown in Fig. 1, and a new state variable Δa is added. The dynamics of the low-pass filter is described by the following differential equation.

$$p(\Delta a) = \frac{1}{T}[p(\Delta\omega) - \Delta a] \tag{21}$$

Eq. (21) is incorporated to the differential equations of each machine (24), resulting in a 5th order model for each machine. Finally, the control signal of Machine i due to PID controller can be rewritten as follows:

$$u_i = K_p \Delta\omega + \frac{K_i}{\omega_o} \Delta\delta + K_d \Delta a \tag{22}$$

$$u_i = K_p \Delta\omega + K'_i \Delta\delta + K_d \Delta a \tag{23}$$

Adding the filter equation, the elemental B and C matrices of (17) are defined as follows:

$$B_i = \begin{bmatrix} 0 & 0 & 0 & \frac{K_{Ei}}{T_{Ei}} & 0 \end{bmatrix}^T, \quad C_i = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad i = 1, \ldots, m \tag{24}$$

The problem of PID-based PSS design is converted to that of an SOF controller design, thanks to the integral relation between ω and δ, and to the inclusion of a low-pass filter. Hence, there is no need for the transformation presented in [28]. The design steps can be summarized as follows:

i. Select an appropriate cut-off frequency for the low-pass-filters on all generating units.
ii. Determine the state-space realization of the linearized power system model around different operating points considering the coupled filters.
iii. Form the polytopic system as given by (5).
iv. Run Algorithm 1 to get an initial positive-definite matrix at all vertices of the polytope determined in step 3. The constrained structure of the matrices P and L should be considered in this algorithm.
v. The initial block-diagonal positive-definite matrix obtained from Algorithm 1 is fed to Algorithm 2 to calculate the SOF gains. The block-diagonal structure of the SOF gain matrix must be preserved as well.

4. Simulation results

The purpose of this section is to demonstrate the merits of the proposed PSS based on a more realistic model. A benchmark model of a two-area four-machine power system [1] is utilized in this study for the following reasons:

1. It is a multimachine system that is accepted in the literature as a tool to study the inter-area mode of oscillations.
2. Each generator is represented by a full seventh-order model that considers stator transients and d–q damper winding. This makes

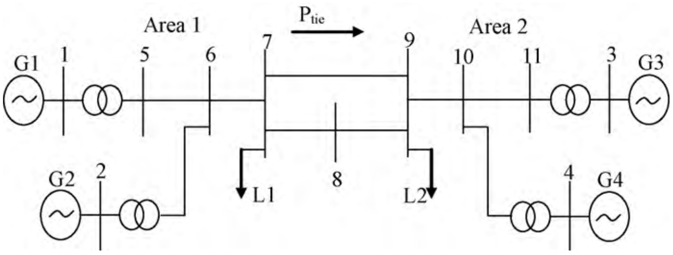


Fig. 2. Two-area four-machine power system [1].

the results reliable when the system is exposed to large disturbance.

3. This model is available as a MATLAB/Simulink demo program [38]. Furthermore, it is equipped with well-tuned power system stabilizers including the standard IEEE-PSS4B [39] and the conventional PSS [1]. This gives credit to the comparison with the proposed PSS.

The benchmark two-area four-machine test power system shown in Fig. 2 is adopted for simulation studies. The test system consists of two fully symmetrical areas linked together by two 230 kV lines of 220 km length. It is specifically designed in [1] to study low frequency electromechanical oscillations in large interconnected power systems. Each area is equipped with two identical round rotor generators rated 20 kV/900 MVA. The synchronous machines have identical parameters except for the inertias which are $H = 6.5$ s in Area 1 and $H = 6.175$ s in Area 2. Thermal plants having identical speed regulators are further assumed at all locations, in addition to fast static exciter with a gain of 200. Saturation limits are imposed on both excitation voltages (E_{fd}) and the supplementary signals (U) as specified in [1]. The loads are represented as constant impedances and spilt between the two areas. The full parameters of a single unit are given in Appendix A.3, while the parameters of the reduced-order model used for design purpose are given in Appendix A.4. The effectiveness of the adopted conventional PSS [1], in damping both local and inter-area oscillations, is assisted by Bode plot analysis as given in Appendix A.5.

The design procedure is initiated by developing a polytopic system. Since damping and frequency of inter-area oscillations depend mainly on the quantity of the tie-line power, the vertex systems of the considered polytope will be calculated at different tie-line powers. These vertex systems are calculated at 200 MW, 400 MW, 600 MW and 700 MW tie-line powers. The load in Area 2 is increased gradually such that steady-state load flow solutions exist at such tie-line powers. The results of load flow solutions are omitted to save space. The linearized models around these operating points are calculated as reported in [1]. The four values of the tie-line power are equivalent to four linearized models. Each model corresponds to a certain operating point and can be seen as a vertex of a polytope; i.e. we construct our polytope based on the given vertices. The proposed design guarantees simultaneous stabilization at the vertices (operating point) as well as any operating point that lies inside the polytope. Throughout the calculation of the linearized model, the dynamics of each machine is represented by a 5th order dynamic model due to the incorporation of a high-pass filter with time constant of 0.01 sec. The input and output matrices of each linearized model, i.e. B and C are the same and having a block-diagonal structure of the elemental matrices (24), while the state matrix A differs from one to another. The polytopic model is fed to Algorithm 1 and an initial block-diagonal positive-definite matrix is found after 2 iterations. The resulting block-diagonal positive-definite matrix (18) comprises the following four matrices on its diagonal entries.

Table 1
Decentralized PID gains.

Machine #	K_P	K_i	K_d
1	47.311	53.077	−0.0267
2	39.426	139.84	0.0697
3	23.285	−17.477	−0.0312
4	21.689	54.573	−0.0796

$$P_1 = \begin{bmatrix} 4.1749 & 280.5 & -3.5076 & -0.0032 & -0.3972 \\ 280.5 & 33341 & -396.6 & -0.5902 & -43.65 \\ -3.5076 & -396.6 & 6.9295 & 0.0082 & 0.8535 \\ -0.0032 & -0.5902 & 0.0082 & 0.0000 & 0.0003 \\ -0.397 & -43.65 & 0.8535 & 0.0003 & 0.6866 \end{bmatrix}$$

$$P_2 = \begin{bmatrix} 5.2445 & 248.49 & -6.2021 & -0.0053 & -0.8559 \\ 248.49 & 19391 & -388.75 & -0.5358 & -52.093 \\ -6.2021 & -388.75 & 11.054 & 0.0111 & 1.6415 \\ -0.0053 & -0.5358 & 0.0111 & 0.0000 & 0.0003 \\ -0.8559 & -52.093 & 1.6415 & 0.0003 & 0.9059 \end{bmatrix}$$

$$P_3 = \begin{bmatrix} 1.5905 & 75.87 & -0.9609 & 0.0000 & -0.1003 \\ 75.87 & 20139 & -365.13 & -0.5159 & -44.696 \\ -0.9609 & -365.13 & 10.821 & 0.0137 & 1.4547 \\ 0.0000 & -0.5159 & 0.0137 & 0.0001 & 0.0008 \\ -0.1003 & -44.696 & 1.4547 & 0.0008 & 0.8298 \end{bmatrix}$$

$$P_4 = \begin{bmatrix} 1.2115 & 43.248 & -1.088 & 0.0001 & -0.1033 \\ 43.248 & 13114 & -391.8 & -0.4972 & -49.987 \\ -1.088 & -391.8 & 17.385 & 0.0198 & 2.3807 \\ 0.0001 & -0.4972 & 0.0198 & 0.0001 & 0.0011 \\ -0.1033 & -49.987 & 2.3807 & 0.0011 & 1.0559 \end{bmatrix}$$

This positive-definite matrix is passed to Algorithm 2 which converges after two iterations and the following SOF gains are obtained and a minimum decay rate of 0.5 is guaranteed, which is satisfactory in power system control topic [11]. The resulting gains of the decentralized PSS are listed in Table 1

Hint: The computational time depends mainly on the number of vertices of the considered polytopic system and the size of the power system, i.e. number of involved machines. A CPU of 1.7 M processor consumes about 15 min to calculate the SOF gains listed in Table 1.

The efficacy of the proposed PID controllers is demonstrated by calculating the closed loop poles of the linearized system at different operating points. A fine grid of operating points is considered by varying the tie-line power between 200 MW and 800 MW and the linearized models of 25 operating points are calculated. The open loop poles of the linearized models around these operating points are calculated and the dominant poles are depicted in Fig. 3 while the dominant closed loop poles are depicted in Fig. 4.

Remarkably, Fig. 3 contains two set of electromechanical oscillations namely, local and inter-area modes of oscillations. A local mode represents the interaction between the machine masses within the same area. Local mode of Area 1 is close to that of Area 2 in terms of damping and frequency; therefore, the two local modes appear in a single cluster. As depicted in Fig. 4, the proposed design guarantees a minimum decay rate that is greater than 0.5. Obviously, Fig. 4 shows the most dominant modes of the controlled system. These modes do not represent the electrometrical oscillations only, i.e. two local modes and one inter-area mode as four clusters appear in the figure.

The zero roots depicted in Fig. 3 are absent in Fig. 4 due to partial state feedback by the incremental rotor angle of each machine in the design procedure, i.e. the integral term in the implemented control law.

Time response simulations are carried out under the following moderate and severe fault scenarios:

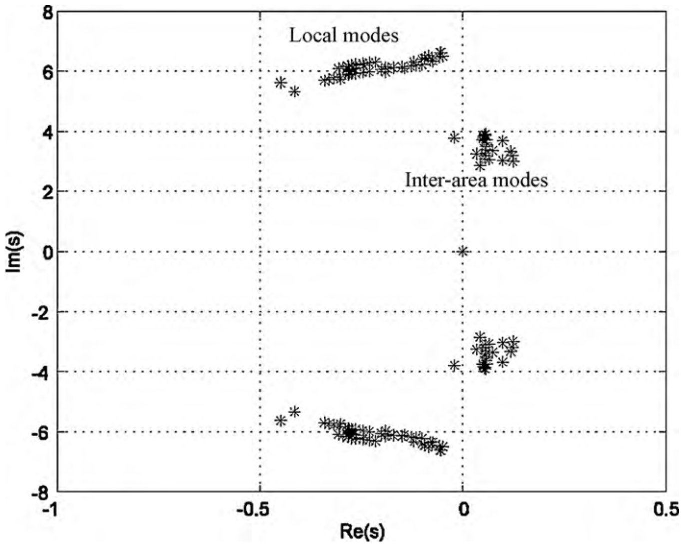


Fig. 3. Dominant open loop poles.

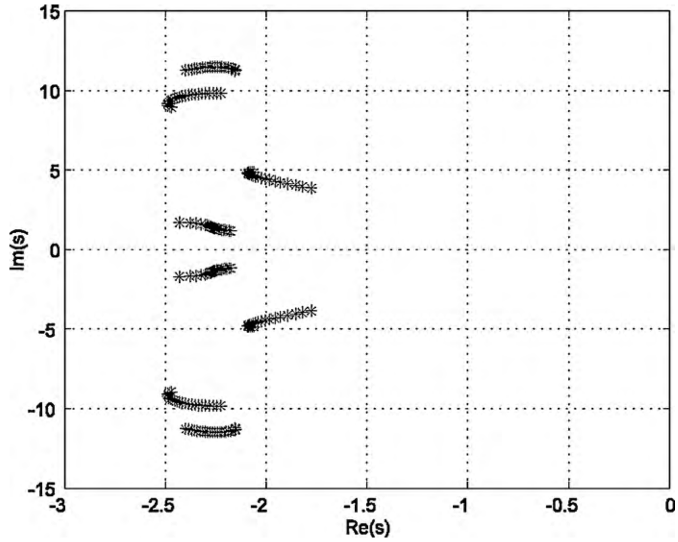


Fig. 4. Dominant poles with the proposed PID-based PSS.

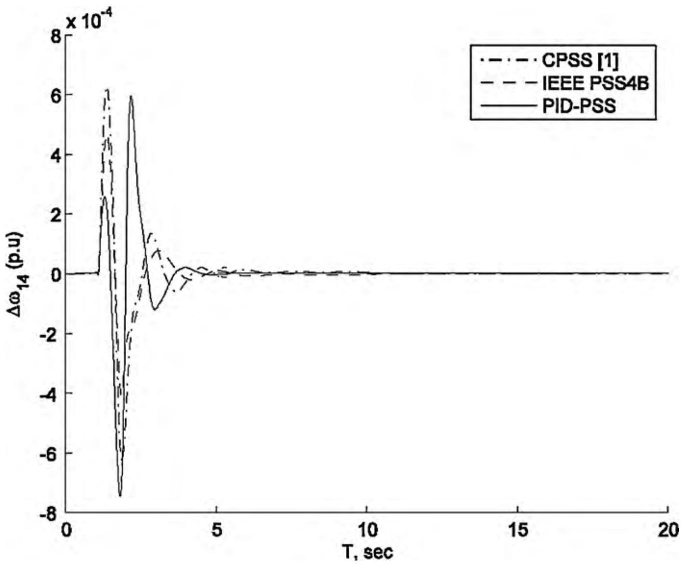


Fig. 5. Relative speed between Machines 1 and 4, subject to Test point I and Fault scenario I.

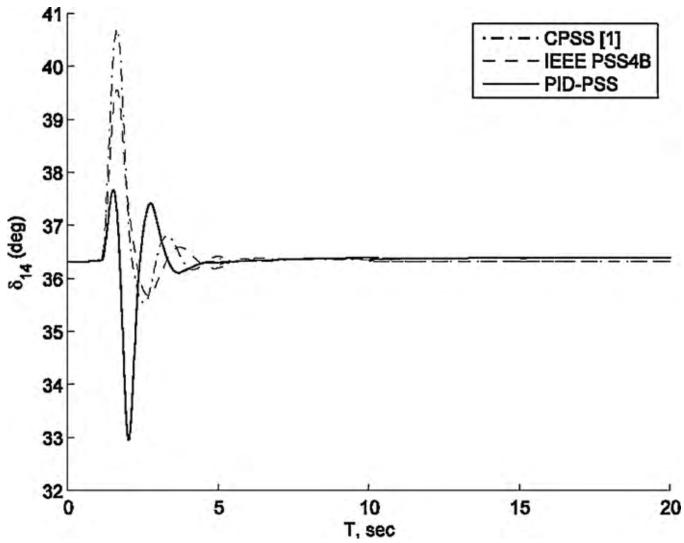


Fig. 6. Relative rotor angle between Machines 1 and 4, subject to Test point I and Fault scenario I.

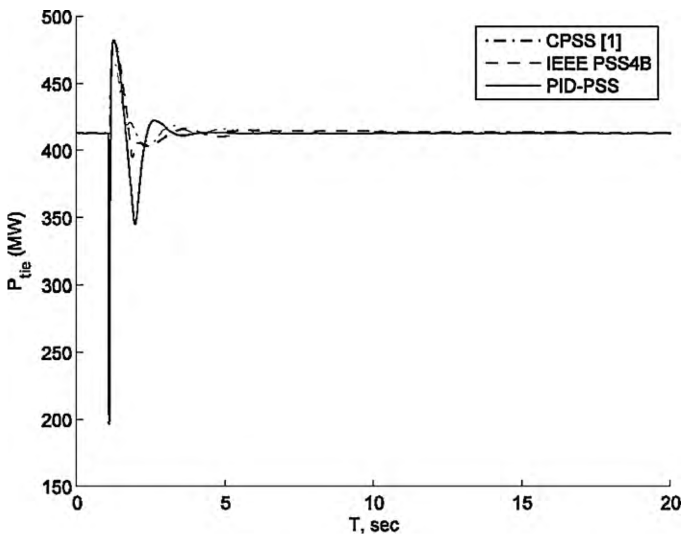


Fig. 7. Tie-line power (MW), subject to Test point I and Fault scenario I. If the system operates at Test point I and undergoes Fault scenario I, the relative speed and angle between Machines 1 and 4 together with tie-line power are depicted in Figs. 5–7 respectively. As seen, the proposed decentralized PID-based PSS achieves good damping characteristics in terms of maximum overshooting and settling time.

1. Fault scenario I consider a moderate fault of a three-phase to ground short circuit at Bus 8 which is self-cleared after 3 cycles.
2. Fault scenario II considers Severe fault of a three-phase to ground short circuit at Bus 8, cleared after 8 cycles by opening the circuit breakers at the ends of the lines connected to this bus.

Also, the following test points are considered throughout simulations:

1. Test point I considers the nominal tie-line power of 413 MW.
2. Test point II considers a tie-line power of 475 MW.
3. Test point III considers a heavy tie-line power of 585 MW.

- If the system operates at Test point I and undergoes Fault scenario I, the relative speed and angle between Machines 1 and 4 together with tie-line power are depicted in Figs. 5–7 respectively. As seen, the proposed decentralized PID-based PSS achieves good

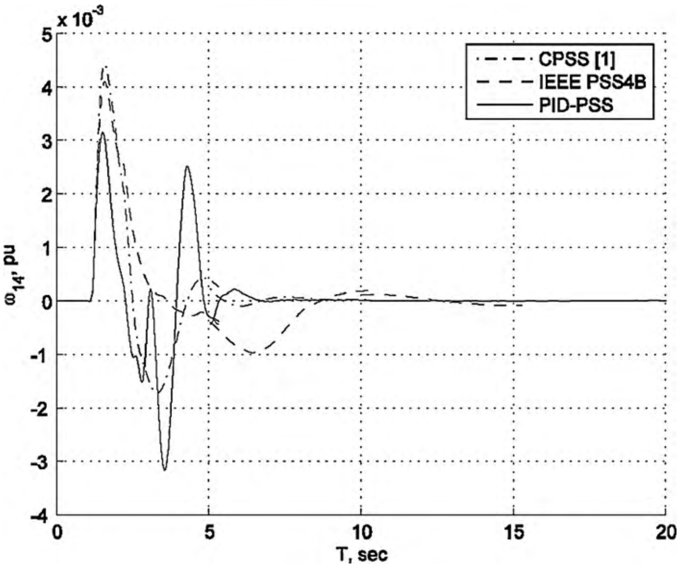


Fig. 8. Relative speed between Machines 1 and 4, subject to Test point I and Fault scenario II.

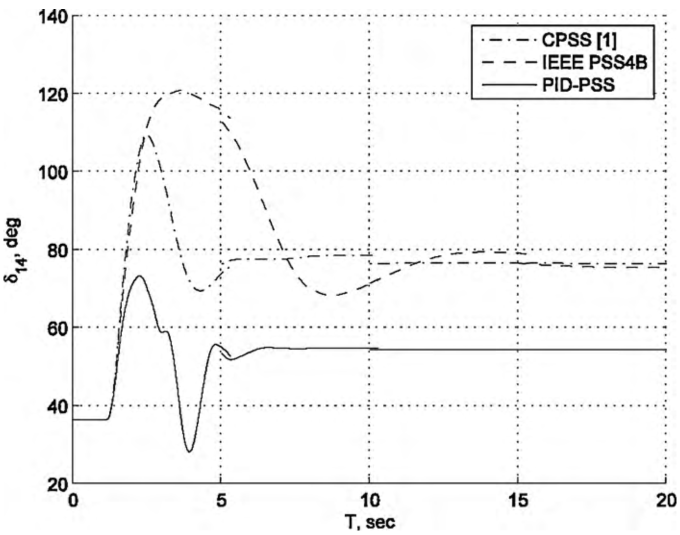


Fig. 9. Relative rotor angle between Machines 1 and 4, subject to Test point I and Fault scenario II.

damping characteristics in terms of maximum overshooting and settling time.

- If the system operates at Test point I and undergoes Fault scenario II, the relative speed and angle between Machines 1 and 4 together with tie-line power are depicted in Figs. 8–10 respectively. Even at nominal tie-line power, the proposed PSS achieves minimum overshooting in relative rotor angle so, it can extend the stability margin of the system. The drift in the operating point between the proposed controller and the standard controllers as shown in Figs. 9 and 10 are mainly due to the steady state value of the control signal provided by the integral action of the proposed PID controller. However, if the integral action is switched-off, the drift in the operating point will disappear as depicted in Fig. 11.
- When the system operates at Test point II and undergoes Fault scenario II, relative speed and relative rotor angle between Machines 1 and 4 together with tie-line power are depicted in Figs. 12–14. Remarkably, the proposed design can handle larger tie-line power, while CPSS and the IEEE-PSS4B fail to maintain system stability at this value of tie-line power.

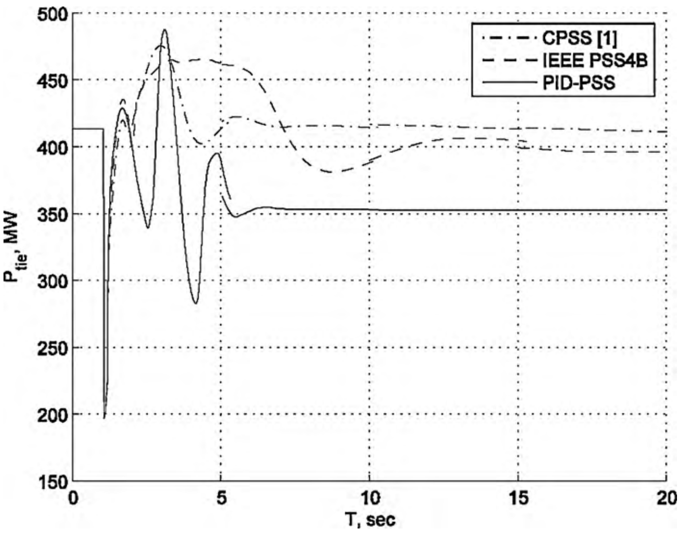


Fig. 10. Tie-line power (MW), subject to Test point I and Fault scenario II.

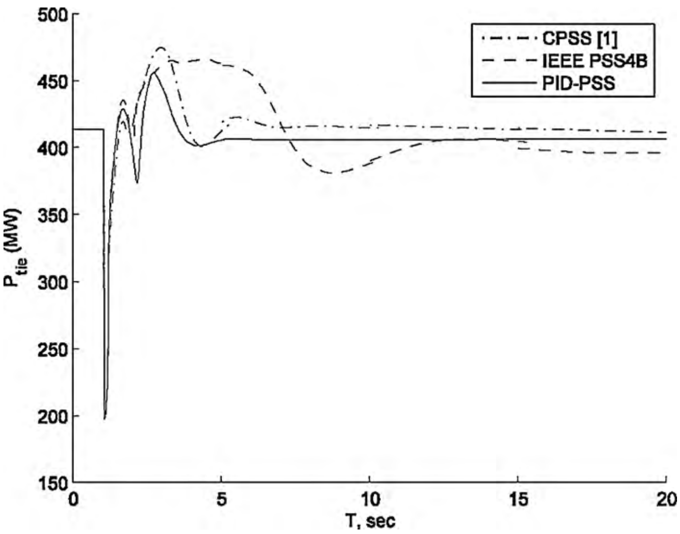


Fig. 11. Tie-line power (MW), subject to Test point I and Fault scenario II with integral action reset.

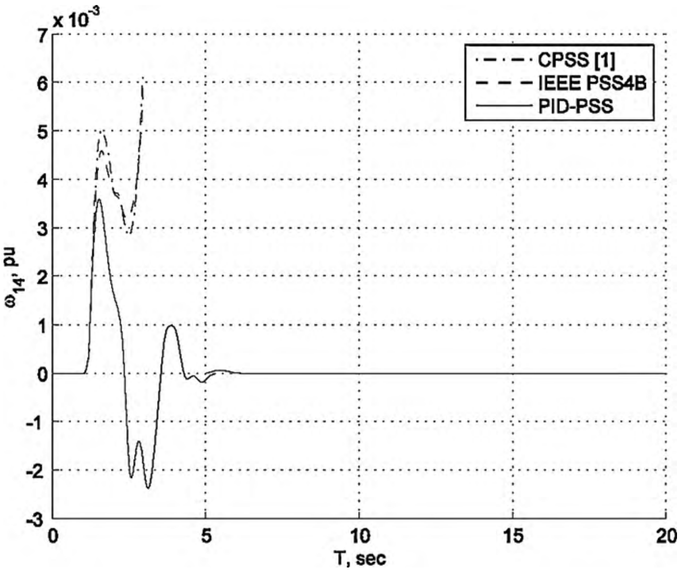


Fig. 12. The relative speed between Machines 1 and 4, subject to Test point II and Fault scenario II.

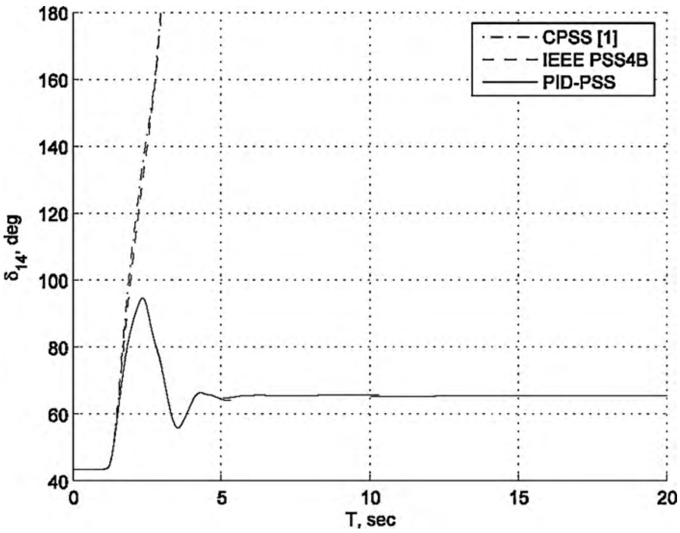


Fig. 13. The relative angle between Machines 1 and 4, subject to Test point II and Fault scenario II.

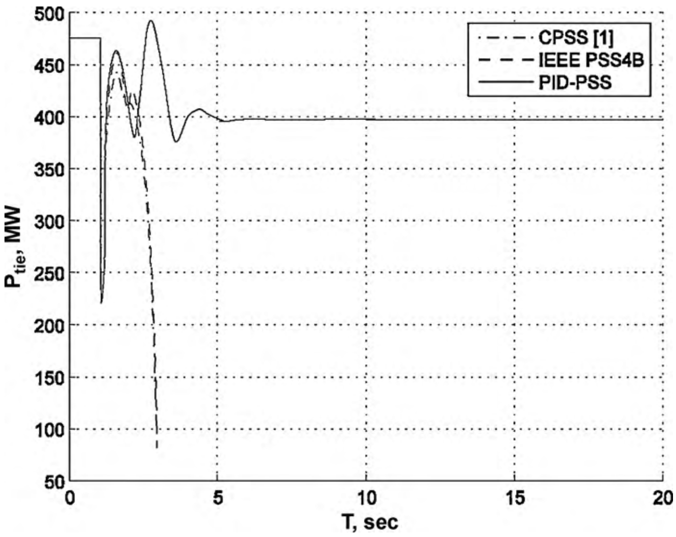


Fig. 14. The tie-line power (MW), subject to Test point II and Fault scenario II.

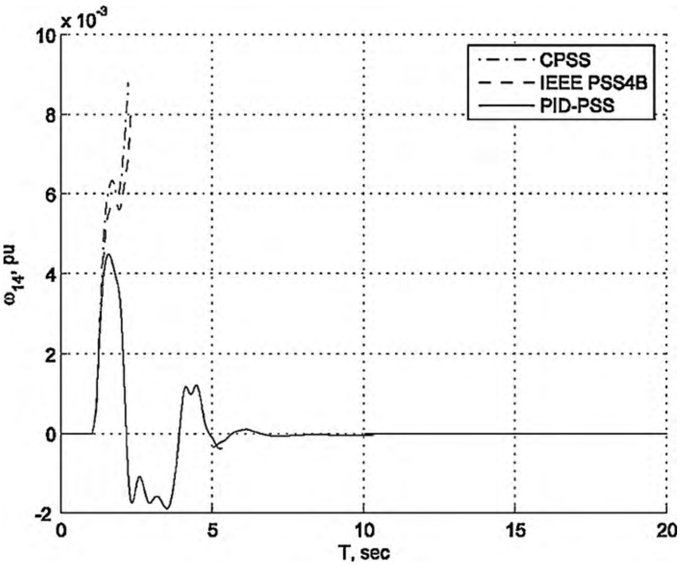


Fig. 15. Relative speed between Machines 1 and 4, subject to Test point II and Fault scenario II.

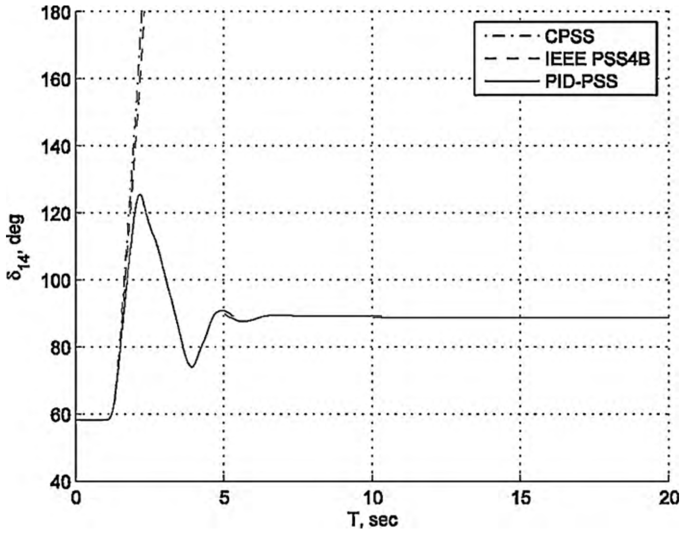


Fig. 16. Relative angle between Machines 1 and 4, subject to Test point II and Fault scenario II.

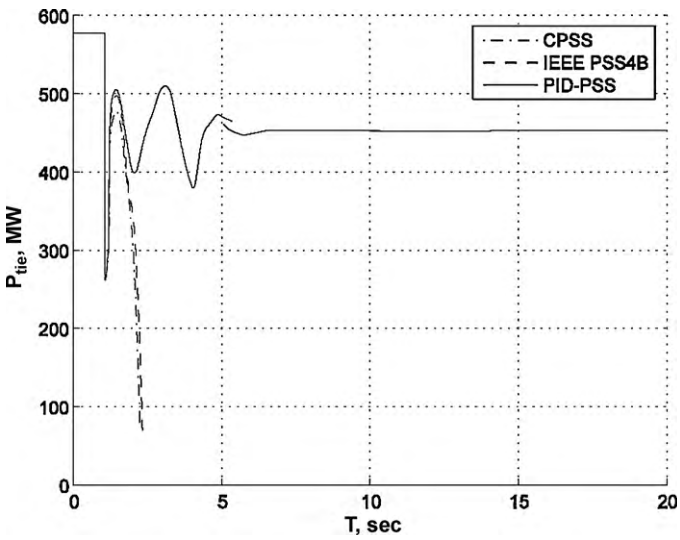


Fig. 17. Tie-line power (MW), subject to Test point III and Fault scenario II.

- When the system operates at Test point III and undergoes Fault scenario II, relative speed and relative rotor angle between Machines 1 and 4 together with tie-line power are depicted in Figs. 15–17 respectively. Obviously, the proposed design is capable to maintain system stability under heavy tie-line power.

5. Conclusion

This paper presents an iterative method, based on LMI techniques, to design a robust decentralized PID-based PSS. The ILMI algorithms reported in [36] have been modified to meet power system specification related to robustness and decentralization of controllers. Sensitivity to sensor noise is also accounted for by including a high-pass filter in the design procedure. The proposed procedure releases the PID-based PSS design from the requirements of invertability of a respective matrix as reported in [28]. Although adoption of a quadratic Lyapunov approach seems to be conservative, the reflection on the values of PID gains is little. However, the design is rather conservative since it provides sufficient stability conditions only. The effectiveness of the proposed design in suppressing both local and inter-area modes of oscillations is verified by a nonlinear simulation of the benchmark model of two-area four-machine power system. Comparing the PID-PSS to a well-tuned conventional PSS and to the standard IEEE-PSS4B affirms the capability of the proposed design to maintain system stability under heavier tie-line powers.

Appendix A.

A.1. The 4th order model adopted to describe each machine dynamics

Differential equations:

$$\begin{aligned}
\dot{\delta}_i &= \omega_o\omega_i \\
\dot{\omega}_i &= \frac{T_{mi}}{M_i} - \frac{E'_{qi}I_{qi}}{M_i} - \frac{(x_{qi}-x'_{di})}{M_i}I_{di}I_{qi} \\
\dot{E}'_{qi} &= -\frac{E'_{qi}}{T'_{doi}} - \frac{(x_{di}-x'_{di})}{T'_{doi}}I_{di} + \frac{E_{fd}}{T'_{doi}} \\
\dot{E}_{fdi} &= -\frac{E_{fdi}}{T_{Ei}} + \frac{K_{Ei}}{T_{Ei}}(V_{refi}+u_i-V_{Ti})
\end{aligned} \tag{25.a}$$

Stator algebraic equations:

$$\begin{aligned}
V_{di} + R_{ai}I_{di} - X_{qi}I_{qi} &= 0 \\
E'_{qi} - V_{qi} - R_{ai}I_{qi} - X'_{di}I_{di} &= 0
\end{aligned} \tag{25.b}$$

A.2. Machine–network transformations [1]

$$\begin{bmatrix} X_{di} \\ X_{qi} \end{bmatrix} = \begin{bmatrix} \sin\delta_i & -\cos\delta_i \\ \cos\delta_i & \sin\delta_i \end{bmatrix} \begin{bmatrix} X_{Di} \\ X_{Qi} \end{bmatrix} \quad i = 1,\ldots,m$$

$$\begin{bmatrix} X_{Di} \\ X_{Qi} \end{bmatrix} = \begin{bmatrix} \sin\delta_i & \cos\delta_i \\ -\cos\delta_i & \sin\delta_i \end{bmatrix} \begin{bmatrix} X_{di} \\ X_{qi} \end{bmatrix} \quad i = 1,\ldots,m$$

where X may be either I or V.

A.3. Full parameters of one unit in the multimachine power system [1]

$R_a = 0.0025$ pu, $x_d = 1.8$ pu, $x_q = 1.7$ pu, $x_l = 0.2$ pu, $x'_d = 0.3$ pu, $x'_q = 0.55$ pu, $x''_d = x''_q = 0.25$ pu, $T'_{do} = 8$ s, $T'_{qo} = 0.4$ s, $T''_{do} = 0.03$ s, $T''_{qo} = 0.05$ s, $H = 6.5$ s, $\omega_o = 377$ rad/s,

Rating $= 900$ MVA, $K_E = 200$, $T_E = 0.001$ s, $E_{fd\min} = 0$, $E_{fd\max} = 12.3$ pu,

$U_{\min} = -0.15$ pu, $U_{\max} = 0.15$ pu.

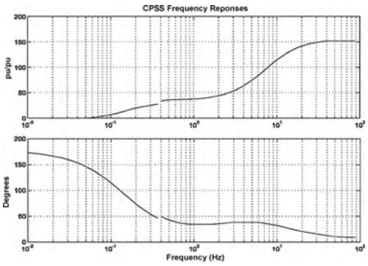
A.4. Parameters of the reduced-order (approximate) model used for the design purpose in Section 4

$x_d = 1.8$ pu, $x_q = 1.7$ pu, $x'_d = 0.3$ pu, $T'_{do} = 8$ s,

$M = 2H = 13$ s, $K_E = 200$, $T_E = 0.001$ s, $\omega_o = 377$ rad/s,

Rating 900 MW

A.5. Bode plot of conventional PSS considering the parameters given in [1].



The above Bode plot verifies that an adequate phase-lead is provided for concerned frequency range of both local and inter-area oscillations which is typically from 0.2 Hz to 3 Hz.

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